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The turbulent dynamo as an instability in a noisy medium

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Abstract. We study an example of instability in presence of a multiplicative noise, namely the spontaneous generation of a magnetic field in a turbulent medium. This so-called turbulent dynamo problem remains challenging, experimentally and theoretically. In this field, the prevailing theory is the Mean-Field Dynamo [1] where the dynamo effect is monitored by the mean magnetic field. In recent years, it has been shown on stochastic oscillators that this type of approach could be misleading. In this paper, we develop a stochastic description of the turbulent dynamo effect which enables us to define unambiguously a threshold for the dynamo effect, namely by globally analyzing the probability density function of the magnetic field instead of a given moment.

PACS. 02.50.-r Probability theory, stochastic processes, and statistics – 47.27.Gs Isotropic turbulence; homogeneous turbulence – 47.27.Jv High-Reynolds-number turbulence

1 Introduction

Classical stability analysis are usually performed in systems where the control parameter is a non-fluctuating quantity, e.g. for laminar flows in hydrodynamics. When the instability occurs in a random system (e.g. a turbulent medium), resulting fluctuation of the control parameter, or multiplicative noise, may generate several surprising effects that have been studied in a variety of systems. The possibility of stabilization by noise has first been evidenced on a Duffing oscillator [2], where the solution $x(t) = 0$ is stable for values of the control parameter above the deterministic threshold. This stabilization is generic for weak intensities of the noise. For stronger intensity, however, noise induced transition may also arise in this system [3]. In the case of a parametric instability, it has been showed experimentally [4] that the instability is sensitive to the bifurcation nature. In the supercritical case, oscillatory bursts (corresponding to a signal with a vanishing mean but a most probable value equal to zero) appear first when the control parameter is increased and are then replaced by a state where the most probable value is no more equal to zero. On the contrary, in the subcritical case, there is coexistence between these two states. This illustrates a central difficulty of instability in presence of multiplicative noise associated with an ambiguity regarding the threshold value, which depends on the definition of the order parameter [5].

The observations and techniques developed in these simple systems may be used to shed new light on some recent issues associated with the dynamo effect, the process of magnetic field generation through the movement of an electrically conducting medium. In this case, the instability results from a competition between amplification of a seed magnetic field via stretching and folding, and magnetic field damping through diffusion. In a laminar fluid, it is controlled by a dimensionless number, the magnetic Reynolds number (Rm), which must exceed some critical value Rm_c for the instability to operate. In a turbulent medium, velocity fluctuations induce fluctuations of the control parameter, making the turbulent dynamo problem similar to an instability in the presence of multiplicative noise. In that respect, recent numerical findings such as observed in [6] may find a natural explanation. In their work, the authors observed short intermittent bursts of magnetic activity separated by relatively long periods, increasing towards the bifurcation threshold. This feature could be explained in terms of a supercritical instability in presence of multiplicative noise since in this case, the bifurcated state is generally composed of oscillatory bursts. More generally, the multiplicative noise paradigm could turn useful to interpret the outcome of recent experiments involving liquid metals. Among the various operating experiments, a clear distinction appears between set up with constrained or unconstrained geometry. In the former case [7], the fluctuation level is very weak. The velocity field is then very close to its laminar (mean) value. In these experiments, dynamos have been observed, at critical magnetic Reynolds number comparable to the theoretical value. In contrast, unconstrained experiments [8] are characterized by a large fluctuation level (as high as 50 per cent). A surrogate laminar Rm_c can then be computed, using the mean velocity field as an input [9] but it is

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not clear whether it will correspond to the actual dynamo threshold, owing to the influence of turbulent fluctuations.

In this paper, we investigate this issue by techniques developed to study the Duffing oscillator and using a stochastic description of small-scale turbulent motions. This subject has been pioneered by Kazantsev [10], Parker [11] and Kraichnan [12], and further developed by the Russian school [13]. It has recently been the subject of a renewed interest, in the framework of anomalous scaling and intermittency [14], or computation of turbulent transport coefficients and probability density functions (PDF) [15].

2 Model

The dynamic of the magnetic field $\mathbf{B}$ in an infinite conducting medium of diffusivity η and velocity $\mathbf{V}$, is governed by the induction equation:

$$\partial_t B_i = -V_k \partial_k B_i + B_k \partial_k V_i + \eta \partial_k \partial_k B_i, \tag{1}$$

with control parameter built using typical velocity and scale as $Rm = LV/\eta$. We decompose the velocity field into a mean part $\bar{V}_i$ and a fluctuating part v_i. In most laboratory experiments, the mean part is provided by the forcing. As such, it is generally composed of large scales, while the fluctuating part collects all short time scales, small-scale movements. In this regard, it is natural to consider the fluctuating part of the velocity as a noise, to be prescribed or computed in a physically plausible manner. The simplest, most widely used shape is the Gaussian, delta-correlated fluctuations, the so-called "Kraichnan's ensemble":

$$\langle v_i(\mathbf{x}, t) v_j(\mathbf{x}', t')\rangle = 2G_{ij}(\mathbf{x}, \mathbf{x}')\delta(t - t'). \tag{2}$$

Equation (1) then takes the shape of a stochastic partial differential equation for B. In that respect, we note that the induction equation is *linear* and does not include explicit back reaction term allowing saturation of any potential growth in the dynamo regime. This back reaction is provided through the velocity which is subject to the Lorentz-Force, a quadratic form of B. Many studies of the dynamo onset adopt a kinematic procedure, where the Lorentz-Force is neglected. This approximation is relevant only for *linear* dynamo mechanisms and cannot be used whenever the dynamo onset is of nonlinear nature. Another problem of the kinematic procedure arises when considering stochastic equations. Indeed, linear stochastic equations renders the threshold determination ill-posed (it leads to a threshold value dependent on the considered moment [3]). For this reason, we prefer to work with a modified induction equation, so as to model this nonlinear back reaction. The back reaction of the magnetic field is indeed responsible for the saturation of the dynamo to a finite value of the magnetic field. This state can be reached by retraction both on the large scales ($\bar{V}_i$) and small scales (v_i) of turbulence. This can influence the value of the growth rate of the magnetic field above the threshold but we strongly believe that it will not play any role in the determination of the threshold value, at least for linear instability mechanism. This is similar to the case of the Duffing oscillator where the threshold is given by the Lyapunov exponent of the *linearized* oscillator [3]. Therefore, the synthetic back reaction we introduce is just a way to avoid magnetic field divergence, not to explore possible nonlinear regimes. Note that we could avoid such a problem by performing a local analysis (see discussion after Eq. (10)) of the Fokker-Planck equation instead of deriving the whole probability distribution.

A practical way of including the effect of the Lorentz force at the onset of the nonlinear regime is to add a saturating term in the induction equation. Symmetry considerations then favor a term like $-cB^2 B_i$. In some sense, this modification is akin to an amplitude equation, and the cubic shape for the non-linear term could be viewed as the only one allowed by the symmetries. Such a procedure has been validated by [16] in the case of the saturation of a Ponomarenko dynamo. Such a cubic form has also been evidenced by Boldyrev [15] by assuming the equality of viscous and dynamical stresses in the Navier-Stokes equation at the onset of backreaction. In the sequel, we show that the precise form of the nonlinear term does not affect the threshold value, which only depends on the behavior for $|B| \to 0$.

A further difficulty is associated with the presence of the diffusive terms. The direct consideration of diffusive terms in the stochastic formulation requires functional derivative and integration, hindering simple analytical description. We propose to model them partially through an additional "molecular" homogeneous noise $\xi_i(t)$, superposed to and uncorrelated from the velocity fluctuation [17], with correlation function $\langle \xi_i(t)\xi_j(t')\rangle = 2\eta\delta_{ij}\delta(t - t')$. In the sequel, we show that this choice provides some sort of saturation for the moments of various order, similar to the role of a viscosity. Damping of magnetic field fluctuations, however, is not properly taken account by this model.

3 Fokker-Planck equation

Using standard techniques [15,18], one can then derive the evolution equation for $P(\mathbf{B}, \mathbf{x}, t)$, the probability of having the field $\mathbf{B}$ at point $\mathbf{x}$ and time t (we assume an homogeneous turbulence for simplicity):

$$\begin{aligned}\partial_t P = &-\bar{V}_k \partial_k P - (\partial_k \bar{V}_i)\partial_{B_i}[B_k P] + \partial_k[\beta_{kl}\partial_l P] \\ &+ c\partial_{B_i}[B^2 B_i P] + 2\partial_{B_i}[B_k \alpha_{lik}\partial_l P] \\ &+ \mu_{ijkl}\partial_{B_i}[B_j \partial_{B_k}(B_l P)],\end{aligned} \tag{3}$$

with the following turbulent tensors:

$$\begin{aligned}\beta_{kl} = \langle v_k v_l\rangle + \eta\delta_{kl}, \quad \alpha_{ijk} = \langle v_i \partial_k v_j\rangle \\ \text{and} \quad \mu_{ijkl} = \langle \partial_j v_i \partial_l v_k\rangle.\end{aligned} \tag{4}$$

Due to incompressibility, the following relations hold: $\alpha^{kii} = \mu^{iikl} = \mu^{ijkk} = 0$.

The physical meaning of these tensors can be found by analogy with the "Mean-Field Dynamo theory" [1,19]. Indeed, consider the equation for the evolution of the mean field, obtained by multiplication of equation (3) by B_i and integration:

$$\begin{aligned}\partial_t\langle B_i\rangle = &-\bar{V}_k\partial_k\langle B_i\rangle + (\partial_k\bar{V}_i)\langle B_k\rangle - 2\alpha_{kil}\partial_k\langle B_l\rangle \\ &+\beta_{kl}\partial_k\partial_l\langle B_i\rangle - c\langle B^2 B_i\rangle. \end{aligned} \quad (5)$$

This equation resembles the classical Mean Field Equation of dynamo theory, with generalized (anisotropic) "α" and "β". The first effect leads to a large scale instability for the mean-field, while the second one is akin to a turbulent diffusivity. A few remarks are in order at this point: i) our mean field equation has been derived without assumption of scale separation. ii) The tensor μ does not appear at this level. In the sequel, it will be shown to govern the stochastic dynamo transition.

For this, we need to identify the threshold as a function of the noise properties. Here, we follow an idea by Mallick and Marcq [3], and focus on the properties of the stationary PDF of the system. Indeed, below the transition, the only stable state is $B = 0$ and the PDF should be a Dirac delta function. Above the transition, other equilibrium states are possible, with non zero magnetic field. However, in the general case, it is not possible to find analytical solution for the equation (3). We thus decompose the magnetic field in its norm and direction. Changing variable $B_i = Be_i$ where e is a unit vector (and can be characterized by $d-1$ angular variables), we can get an equation for $P(B, e_i, x) = JP(B_i)$ where $J = B^{d-1}$ is the Jacobian of the transformation. One should note that $\langle B\rangle$ is a suitable order parameter contrary to $\langle B_i\rangle$, which can be null even above the dynamo threshold (e.g. for a rotating magnetic field with a constant norm). We now assume that there is an uncoupling for P as $P(B, e_i) = P(B)G(e_i, x)$, and perform an average over the angular variables, to find a closed equation for $P(B)$. In some sense, this can be regarded as a mean-field argument where we average over the fast angular variables. This argument cannot be proved in the general case but it has been checked in the case of a Duffing oscillator [3]. The final equation for $P(B)$ becomes:

$$\frac{\partial P}{\partial t} = a\frac{\partial}{\partial B}\left[B\frac{\partial}{\partial B}(BP)\right] - b\frac{\partial}{\partial B}(BP) + c\frac{\partial}{\partial B}(B^3P), \quad (6)$$

where the coefficients are given by averages over the position and the angular variables $\langle\bullet\rangle_\phi = \int \bullet\, G(\mathbf{e},\mathbf{x})\, d\mathbf{x}d\mathbf{e}$:

$$\begin{aligned} a &= \langle\mu_{ijkl}e_ie_je_ke_l\rangle_\phi \\ b &= \langle\partial_k\bar{V}_ie_ie_k\rangle_\phi + \langle\mu_{ijkl}(\Delta_{ik}e_je_l + \Delta_{kj}e_ie_l)\rangle_\phi, \end{aligned} \quad (7)$$

where we used $\Delta_{ij} = \partial_{e_i}(e_j) = \delta_{ij} - e_ie_j$ an "angular Dirac tensor". One can notice that these coefficients only explicitly involve the tensor μ. Nevertheless, one must keep in mind that the tensor α and β enter these expressions by mean of the angular distribution $G(e_i, x)$, whose expression involves this two tensors in the general case. In the derivation of equation (6), we use the following expression for the gradient with respect to the magnetic field:

$$\frac{\partial}{\partial B_i}(B_kG) = e_ie_k\frac{\partial}{\partial B}(B\,G) + \frac{\partial}{\partial, e_i}(e_kG), \quad (8)$$

where the first part contains derivatives only with respect to the radial variable and the last one with respect to the angular ones. Using this decomposition in equation (3), integrating with respect to $\mathbf{x}$ and $\mathbf{e}$ and making use of the "integration by part on the angular variable" formula:

$$\begin{aligned}\langle F(\mathbf{e})\partial_{e_i}[e_jG(\mathbf{e})]\rangle_\phi = &(d-1)\langle e_ie_kF(\mathbf{e})G(\mathbf{e})\rangle_\phi \\ &-\langle\partial_{e_i}[F(\mathbf{e})]e_jG(\mathbf{e})\rangle_\phi,\end{aligned} \quad (9)$$

leads to equation (6).

4 The dynamo threshold

An obvious stationary solution of (6) is a Dirac function, representing a solution with vanishing magnetic field. Another stationary solution can be found by setting $\partial_t P = 0$ in (6), with solution:

$$P(B) = \frac{1}{Z}B^{b/a-1}\exp\Big[-\frac{c}{2a}B^2\Big], \quad (10)$$

where Z is a normalization constant. This solution can represent a meaningful probability density function only if it can be normalized. This remark provides us with a bifurcation threshold: there is dynamo whenever (10) is integrable, i.e., when solution other than vanishing magnetic field are possible. Let's us now comment again on the choice we made for the particular form of the back reaction. Instead of finding a particular form for the probability density function, we could have performed a local analysis of equation (6). For $B \ll 1$, it is obvious that the important terms in the right hand side are the one involving a and b, which leads to $P(B) \propto B^{b/a-1}$ for $B \ll 1$ irrespectively of the particular form of the saturating term (if it is negligible compared to a linear term). The condition of existence for this solution we will discuss now is thus the same as that for equation (10).

Condition of integrability at infinity of (10) requires a be positive. This illustrates the importance of the non-linear term which is essential to ensure vanishing of the probability density at infinity. Condition of integrability near zero requires b/a be positive. This leads us to identify a necessary and sufficient condition for existence of a stationary dynamo as

$$a > 0 \quad \text{and} \quad \frac{b}{a} > 0 \qquad DYNAMO. \quad (11)$$

In some sense, this bifurcation (I) is obtained using the mean field as control parameter. Another bifurcation threshold can be defined using the most probable field as control parameter. Indeed, an elementary calculation shows that the condition for a maximum in the PDF is $b > a$. Therefore, the bifurcation threshold (II) with the most probable field as control parameter is defined by $b = a$. This difference may have some relevance when analyzing real data from experiment.

To get some physical insight on the nature of this two bifurcations, we performed simulations of a 1 dimensional (1D) non-linear stochastic system:

$$\partial_t x = [b + \xi(t)]x - \gamma x^3 \quad (12)$$

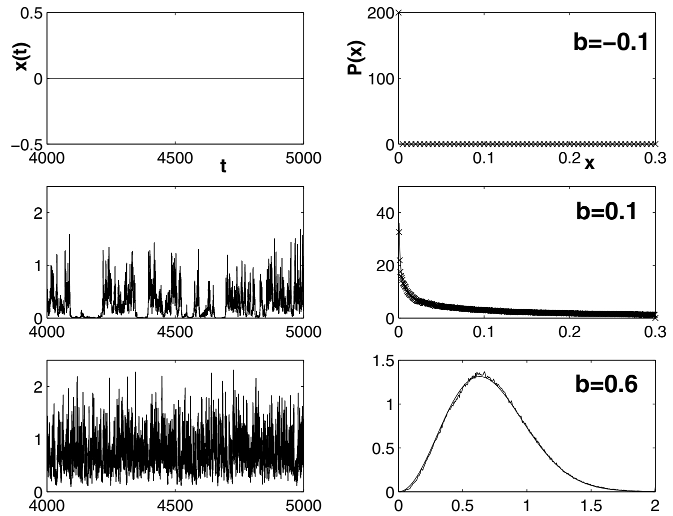

Fig. 1. Result of the surrogate 1D model (12): On the left side we show time series for $a = 0.2$, $\gamma = 1$ and 3 different values of the parameter b. On the right side, the corresponding PDF and the theoretical curve corresponding to equation (10).

with $\langle \xi(t)\xi(t')\rangle = 2a\delta(t - t')$. Even if this model cannot be seen as 1D version of our dynamo problem (there is no dynamo effect for dimension lower than 2), they share some common feature: they both have deterministic and stochastic multiplicative excitations and damping. Furthermore, it may be checked that the stationary PDF in this case is exactly given by equation (10). The great advantage of model (12) is that it can be easily solved numerically. Therefore, we hope that the time series and associated PDF are good illustration of the output of our 3D model, and the meaning of the two bifurcations discussed above. The time series and PDF for three different values of the control parameter are shown in Figure 1. The simulations show that the bifurcation (I) leads to an intermittent behavior for the magnetic field reminiscent of the characteristic behavior of instability in presence of multiplicative noise: typically, the magnetic energy exhibits bursts separated by long quiescent (zero magnetic energy) period. On the contrary, the bifurcation (II) is quite different in nature because of a well defined mean value for the magnetic field and fluctuations around this mean. The same type of behavior (transition to an intermittent and fluctuating dynamo) has also been reported for a 2D model of a solid dynamo in presence of multiplicative noise [20] and we thus expect that it is reminiscent of a bifurcation with multiplicative noise.

Equation (10) cannot rigorously capture these intermittent states. However, two facts are very suggestive of such a type of bifurcation in our solution: (a) the distribution looks like a pure fluctuation distribution, with ill-defined mean value; (b) the scaling for $\|B\| \ll 1$ (magnetic energy) is the same as that of [6]: $P(\|B\|) = \|B\|^\gamma$ with $\gamma = b/D - 1$, where D is a "diffusion coefficient" for the finite-time Lyapunov exponent. Note also that if we consider the dynamo instability in absence of noise, $a = 0$, the two bifurcation threshold collapse. As stated by [6], the bifurcation corresponding to $b > 0$ may be difficult to observe in real experiments because, under the threshold, the presence of the Earth external magnetic field always gives rise to magnetic fluctuations qualitatively similar to that above the threshold (magnetic bursts separated by quiescent period). This effect can be taken into account by adding an additive noise to equation (1).

In the theory of dynamical systems stability, the instability criterion is usually associated with the existence of a positive Lyapunov exponent for the growth of the system energy: $\lim_{t\to\infty} \ln B^2/2t = \lim_{t\to\infty} \ln B/t$. It is possible to find this exponent, by multiplying equation (6) by $\ln B$ and integrating with respect to B. This yields $\partial_t\langle \ln B\rangle = b$, meaning that the Lyapunov exponent in our system is equal to b. The two instability criteria (existence of a normalizable solution or a positive Lyapunov exponent) are therefore identical provided $a > 0$, a necessary condition for integrability of the PDF at infinity.

5 Discussion

5.1 Diffusive effects

Our model does not take into account diffusive effects. Their influence has been studied using other techniques, such as random matrices [21] or variational principles [22]. In these contributions, two mechanisms of diffusive actions have been evidenced, which find a counterpart in our noisy system. It is therefore interesting to recall them now.

In the absence of any diffusion, magnetic field growth is controlled by the stretching rate, namely the largest eigenvalue of $S_{ij} = (1/2)(\partial_j V_i + \partial_i V_j)$. Any initial magnetic field fluctuation will then grow and orientate itself in the direction of the stretching rate. When diffusivity is present, new phenomena occur which may counteract or even cancel this growing mechanism. The dynamo threshold is then determined by balance between these impeding mechanisms and the growth induced by the stretching. First, diffusive action orientates asymptotically in time any solenoidal field such as the magnetic field along the contracting direction, namely the smallest (negative) eigenvalue of S_{ij} [21]. The limit induced by this process (and then the dynamo threshold) is independent of the diffusivity amplitude. Second, diffusivity damps the magnetic field growth by an amount proportional to the diffusive transport coefficient. The balance between this damping and the kinematic growth results in a critical magnetic Reynolds number, below which no dynamo is possible [22].

5.2 Qualitative influence of noise

In the light of these results, it is now interesting to discuss qualitatively the meaning of our main result (11). It is possible to show that for isotropic or axisymmetric velocity fluctuations, the coefficient a is positive. So we suspect that the main condition for existence of a dynamo is positivity of b. This coefficient is actually built from two terms, that behave very differently through noise action.

The first term, $b_1 = \langle \partial_k \bar{V}_i e_i e_k \rangle_\phi$ is not directly proportional to noise intensity. The action of noise on that term is mainly through vector orientation. In the absence of noise, the magnetic field mainly grows in the direction given by the largest eigenvalue of $\bar{S}_{ij} = \partial_j \bar{V}_i$. Consider now a situation where one increases the noise level. This noise causes arbitrary magnetic field orientation and may even induce changes in the distribution of magnetic field orientation through systematic effect. Therefore, b_1 will decrease with respect to its deterministic value. For example, if noise induces a flat distribution for e_i, then $b_1 = \bar{S}_{ii} = 0$. Even more dramatic results can be obtained if the noise tends to align the vector along a direction of negative eigenvalue for S_{ij}, since in that case the factor b_1 becomes negative, with a lower bound given by the minimal (and negative) eigenvalue of S_{ij}. The action of noise through this mechanism is therefore necessarily negative for dynamo action, but bounded: it may not be increased to infinity by increasing the noise intensity.

The second term of the Lyapunov $b_2 = \langle \mu_{ijkl}(\Delta_{ik} e_j e_l + \Delta_{kj} e_i e_l) \rangle_\phi$ is proportional to μ and can actually be shown to be positive for isotropic or axisymmetric velocity fluctuations. So, we believe that in most circumstances, this term will tend to drive the Lyapunov towards positive values, i.e. to favor dynamo action. Contrarily to the orientation mechanism, this additive effect is proportional to the noise intensity, and can grow without limit: it can be made as large as possible by increasing the noise intensity. On that basis, we expect that, as the noise is slowly increased, this effect goes from sub-dominant to dominant in the determination of the Lyapunov exponent. One can also notice that the tensor μ is proportional to the gradient of the fluctuating part of the velocity field (the turbulence). Consequently, it will be much more important for small-scale motions compared to large scale ones. We can thus conclude that small (fluctuating) scales of turbulence are probably very important for the process of magnetic field generation through dynamo action.

Increasing the noise intensity, one should then first observe the (negative) orientation mechanism, i.e. a threshold augmentation, followed by the (positive) additive effect, resulting in a decrease of the dynamo threshold. This qualitative behavior is depicted in Figure 2. Such a scenario is actually observed in another stochastic system with multiplicative noise, the Duffing oscillator.

5.3 Comparison with mean field theory

It is important to notice that our approach determines the dynamo threshold by means of the whole probability density function of the magnetic field and not solely by the study of the growth rate of one particular moment (as it is the case in Mean Field Theory where one concentrates for example on the growth of the mean magnetic field). This necessity has already been evidenced in the context of shell models of turbulence [23] where the intermittent behavior of the magnetic field prevents the description of the late stage of MHD turbulence by average quantities.

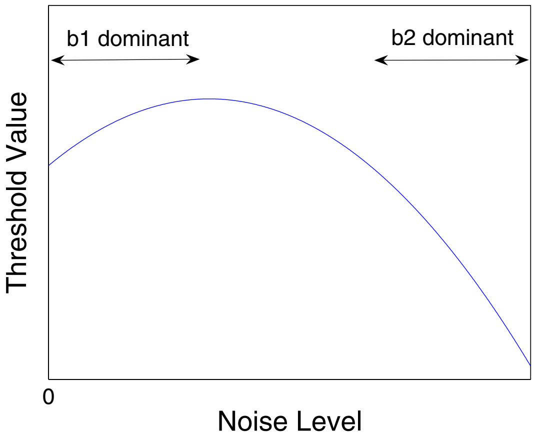

Fig. 2. Schematic diagram of the influence of increasing the noise intensity on the location of the dynamo threshold. For a weak noise, there is a stabilization of the dynamo instability whereas for strong noise, instability occurs for lower value of the control parameter.

Moreover, our approach gives a quantitative criterion on the dynamo threshold (namely $b > 0$). However, its practical implementation requires the measure on the angular and position variables $G(\mathbf{x}, \mathbf{n}, t)$ and the average of the tensor μ with this measure. One can obtain the measure by integrating equation (3) with respect to B. Unfortunately the equation for G can not be solved in the general case and particular types of turbulence statistics have to be considered (isotropic, axisymmetric, etc...). Work is under progress to determine the angular measure in these simple cases. It is however interesting to note that this measure explicitly involves the tensors α and β defined in (4) and appearing in the Mean Field Equations (5). In that sense, the dynamo threshold depends on these tensors, albeit in a less explicit way than in the Mean Field Equations (MFE). It would therefore be interesting to confront threshold derived from (MFE), which are μ independent, and from our theory, to see what kind of error in the threshold determination one can expect by using MFE instead of the true, non-perturbative theory.

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